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DESIGN OF LAMINATED TORSION BAR SPRINGS

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This report provides an analytical approach to the design of torsion leaf spring packs for use in cannon breech mechanisms, vehicle suspensions, etc. The design procedure presents an expeditious simple method for determining spring pack dimensions when parameters such as total required torque, spring pack angle of twist, free length of leaf spring, maximum tensile working stress, and maximum torsional working stress are known. The unknown parameters to be determined are leaf spring thickness, leaf spring width, and the number of leaf springs
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20. Abstract (CONT'D)

required per pack to provide the required total torque. Of interest is the fact that leaf spring thickness is dependent solely on maximum shearing stress, angle of twist per unit length, and shear modulus of elasticity. Also, leaf spring width is dependent solely on maximum tensile stress, angle of twist per unit length, and tensile modulus of elasticity.

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NOTATIONS

α	Coefficient depending on ratio b/c.
β	Coefficient depending on ratio b/c.
θ	Angle of twist per unit length, Radians/inch.
ϕ	Angle of twist, degrees.
σ_1	Maximum principal tensile stress, psi.
σ_2	Maximum principal compressive stress, psi.
σ_A	Compressive stress at mid-point of width "b", psi.
σ_B	Tensile stress at both ends of width "b", psi.
σ_W	Tensile working stress, psi.
σ_Y	Tensile yield stress, psi.
σ_Z	Tensile or compressive stress at distance "y", psi.
σ_U	Ultimate tensile stress, psi.
τ_A	Maximum shearing stress at mid-point of width "b", psi.
τ_M	Maximum combined shearing stress, psi.
τ_P	Shearing stress at distance "y", psi.
τ_W	Torsional working stress, psi.
τ_Y	Torsional yield stress, psi.
τ_{ZY}	Shearing stress at distance "y", psi.
b	Width of rectangular strip, inches.
c	Thickness of rectangular strip, inches.
ℓ	Free length of rectangular strip, inches.
y	Distance from Z-axis, inches.

n	Number of leaf springs per pack.
E	Tensile modulus of elasticity, psi.
G	Shear modulus of elasticity, psi.
H	Total height of laminated spring pack, inches.
S.F.	Safety factor.
T	Total torque per laminated spring pack, in-lbs.
T_1	Torque due to torsional shear, in-lbs.
T_2	Torque due to longitudinal stress, in-lbs.
T_n	Torque per leaf spring, in-lbs.

INTRODUCTION

The Saint Venant's analysis indicates that when a flat strip is twisted by applying moments at its ends, a plane transverse section therewithin does not remain a plane section after twisting. Said section becomes a warped surface which effectuates an increase in shearing stress in some parts of the section and a decrease in other parts when compared to stresses that would occur if the section did not warp. The center of the strip's long side will experience maximum shearing stress, and the corners will experience zero shearing stress. Also, due to said twisting, the longitudinal edges form a helix whose fibers become longer than the center fibers. Simultaneously, said twisting causes the strip to shorten axially. Therefore, the net effect is the introduction of tensile stresses in the outer fibers, and compressive stresses within the central fibers. These longitudinal secondary stresses introduce a secondary torque which supplements the torque associated with the torsional shearing stresses. It should be noted that these stresses prevail at a sufficient distance from the strip ends, without consideration as to the manner in which the strip's ends are secured.

When a pack of flat strip springs are twisted while secured within rectangular type end restraints, localized stresses develop where said strips emerge from the end restraints. Depending on the particular type of end restraint design, stress of considerable magnitude can exist at the juncture to cause fatigue failure. Since sections perpendicular to the strip spring's torsional axis warp when twisted, the strip ends should be allowed to warp freely within the rectangular cavity of the end restraints in order to minimize operational stresses. Therefore, strip spring working stresses should be established with due consideration given to the type of end restraint design used.

Due to friction between unlubricated strips of a laminated torsion bar spring pack during twisting, hysteresis will prevail between the loading and unloading action of the spring pack. To reduce inter-strip friction damping, liquid or solid film lubricant may be used.

THEORETICAL ANALYSIS

The analytical treatment of solid noncircular members in torsion, as presented herein, is based on the following assumptions:

(1) The strip is straight, of uniform rectangular section, and of homogeneous isotropic material.

(2) The strip is loaded only by equal and opposite twisting couples which are applied at its ends in planes normal to its longitudinal axis.

(3) The strip is stressed within the elastic limit of the material.

(4) The condition of loading is such that the end sections of the strip are free to warp within the end restraints.

Torque Due To Torsional Shear

$$T_1 = \alpha b c^2 \tau_A$$

$$T_1 = \beta b c^3 G\theta$$

Data indicates that α and β are approximately equal to 1/3 for $b/c \geqq 10$.

$$T_1 = \frac{1}{3} b c^3 G\theta \tag{1}$$

$$T_1 = \frac{1}{3} b c^2 \tau_A \tag{2}$$

Thickness of Rectangular Strip

Equating Eqs. (1) and (2) and solving for "c".

$$c = \frac{\tau_A}{G\theta} \tag{3}$$

Longitudinal Stress Distribution in Rectangular Strip

$$\sigma_Z = \frac{E\theta^2}{b}\left[y^2 - \frac{b^2}{12}\right] \tag{4}$$

When $\sigma_z = 0$

$$y = \sqrt{\frac{b^2}{12}} \tag{5}$$

When $y = 0$

$$\sigma_A = -\frac{E\theta^2 b^2}{24} \quad \text{(Max. Compressive Stress)} \tag{6}$$

When $y = \pm\, b/2$

$$\sigma_B = \frac{E\theta^2 b^2}{12} \quad \text{(Max. Tensile Stress)} \tag{7}$$

$$\sigma_B = -2\sigma_A \tag{8}$$

Solving Eqs. (6) and (7) for "b^2"

$$b^2 = -\frac{24\sigma_A}{E\theta^2} \tag{9}$$

$$b^2 = \frac{12\sigma_B}{E\theta^2} \tag{10}$$

Torque Due to Longitudinal Stress

$$T_2 = \frac{1}{360} Ecb^5\theta^3 \tag{11}$$

Total Twisting Torque Per Strip Spring

Combine Eqs. (1) and (11)

$$T_n = T_1 + T_2$$

$$T_n = \frac{1}{3} bc^3 G\theta + \frac{1}{360} Ecb^5\theta^3 \tag{12}$$

Number of Strip Springs Required Per Pack

Neglecting inter-spring friction between strip springs of equal width and thickness, it has been determined that the developed total torque is essentially that of a single strip spring multiplied by the number of strip springs in the pack.

$$n = \frac{T}{T_n} \tag{13}$$

Total Height of Laminated Spring Pack

$$H = nc \tag{14}$$

Shearing Stress Distribution in Rectangular Strip

With reference to Fig. 2, the maximum shearing stress "τ_A" exists at point "A", the center of side "b". Assume the shearing stress to vary from "A" to "B" approximately as the ordinates of a parabola. Therefore, the stresses along AB are represented by the ordinates of the portion HMB of a parabola.

Let AH represent "τ_A", and MP represent "τ_P"

$$AH = BK = \tau_A$$

$$PM = \tau_P$$

Equation of Parabola:

$$y^2 = kx'$$

When $y = b/2$; $x' = BK$

$$(b/2)^2 = k(BK)$$

$$k = \frac{(b/2)^2}{BK}$$

When $y = HN$; $x' = (BK-MP) = NM$

$$y^2 = \frac{(b/2)^2}{BK} x'$$

$$y^2 = \frac{(b/2)^2}{BK} (BK-MP)$$

$$\frac{BK-MP}{BK} = \frac{y^2}{(b/2)^2}$$

$$1 - \frac{MP}{BK} = \left[\frac{2y}{b}\right]^2$$

$$1 - \frac{\tau_P}{\tau_A} = \left[\frac{2y}{b}\right]^2$$

$$\tau_P = \tau_A\left[1 - \left(\frac{2y}{b}\right)^2\right] \tag{15}$$

Combination of Longitudinal and Shearing Stresses

With reference to Fig. 3, note that the portion of the parabola AP'B' represents the portion of the parabola HMB shown in Fig. 2.

(a) At point "A", the longitudinal stress "σ_A" is maximum compressive, and the shearing stress is "τ_A".

$$\sigma_1 = \frac{\sigma_A}{2} + \sqrt{\left[\frac{\sigma_A}{2}\right]^2 + \tau_A{}^2} \quad \text{(Max. Principal Tensile Stress)} \tag{16}$$

$$\sigma_2 = \frac{\sigma_A}{2} - \sqrt{\left[\frac{\sigma_A}{2}\right]^2 + \tau_A{}^2} \quad \text{(Max. Principal Compressive Stress)} \tag{17}$$

$$\tau_M = \sqrt{\left[\frac{\sigma_A}{2}\right]^2 + \tau_A{}^2} \quad \text{(Max. Combined Shearing Stress)} \tag{18}$$

(b) At point "P", the longitudinal stress is zero, and the shearing stress is "τ_P".

$$\sigma_1 = \tau_P \quad \text{(Max. Principal Tensile Stress)} \tag{19}$$

$$\sigma_2 = -\tau_P \quad \text{(Max. Principal Compressive Stress)} \tag{20}$$

$$\tau_M = \tau_P \quad \text{(Max. Combined Shearing Stress)} \tag{21}$$

(c) At point "B", the longitudinal stress "σ_B" is maximum tensile, and the shearing stress "τ_B" is zero.

$$\sigma_1 = \sigma_B \quad \text{(Max. Principal Tensile Stress)} \tag{22}$$

$$\sigma_2 = 0 \quad \text{(Max. Principal Compressive Stress)} \tag{23}$$

$$\tau_M = \sigma_B/2 \quad \text{(Max. Combined Shearing Stress)} \tag{24}$$

EXAMPLE PROBLEM AND SOLUTION

Given: $T = 3600$ in-lbs. $\qquad G = 11.5 \times 10^6$ psi

$\phi = 135°$ $\qquad E = 30 \times 10^6$ psi

$\ell = 11.75"$ $\qquad \sigma_U = 256{,}000$ psi

$\theta = .2005$ Rad./in.

Material: A1S1 1095 Clock Spring Steel

Find: c, b, T_n, n, H, σ_1, σ_2, and τ_M

Tensile Yield Stress

$$\sigma_Y = .85\ \sigma_U = .85 \times 256{,}000 = 217{,}600 \text{ psi}$$

Compressive Yield Stress

$$\sigma_Y = 392{,}000 \text{ psi}$$

Torsional Yield Stress

$$\tau_Y = .6\ \sigma_Y = .6 \times 217{,}600 = 130{,}560 \text{ psi}$$

Tensile Working Stress (At Pt. B)

$$\sigma_W = \sigma_B = \frac{\sigma_Y}{S.F.} = \frac{217{,}600}{1.38} = 157{,}681 \text{ psi}$$

Maximum Combined Shearing Stress (At Pt. A)

$$\tau_M = \frac{\tau_Y}{S.F.} = \frac{130{,}560}{1.8} = 72{,}533 \text{ psi}$$

From Eq. (8):

$$\sigma_A = -\frac{\sigma_B}{2} = -\frac{157{,}681}{2} = -78{,}840 \text{ psi}$$

From Eq. (18):

$$\tau_A^{\,2} = \tau_M^{\,2} - \left[\frac{\sigma_A}{2}\right]^2 = (72{,}533)^2 - \left[\frac{-78{,}840}{2}\right]^2$$

$$\tau_A = 57{,}338 \text{ psi}$$

From Eq. (3):

$$c = \frac{\tau_A}{G\theta} \quad \frac{57{,}338}{11.5 \times 10^6 \times .2005}$$

$$c = .0249$$

Let $$c = .025''$$

From Eq. (10):

$$b^2 = \frac{12\ \sigma_B}{E\theta^2} \quad \frac{12(157,681)}{30 \times 10^6 (.2005)^2}$$

$$b = 1.2526$$

Let $$b = 1.250''$$

From Eq. (12):

$$T_n = \frac{1}{3} bc^3 G\theta + \frac{1}{360} Ecb^5\theta^3$$

$$T_n = \frac{1.250(.025)^3 11.5 \times 10^6 \times .2005}{3} + \frac{30 \times 10^6 \times .025(1.25)^5(.2005)^3}{360}$$

$$T_n = 66.2564 \text{ in-lbs.}$$

From Eq. (13):

$$n = \frac{T}{T_n} = \frac{3600}{66.2564} = 54.3343$$

Let $$n = 54$$

From Eq. (14):

$$H = nc = 54 \times .025$$

$$H = 1.350''$$

From Eq. (16):

$$\sigma_1 = \frac{\sigma_A}{2} + \sqrt{\left[\frac{\sigma_A}{2}\right]^2 + \tau_A{}^2}$$

$$\sigma_1 = \frac{-78,840}{2} + \sqrt{\left[\frac{-78,840}{2}\right]^2 + (57,338)^2}$$

$$\sigma_1 = 30,161 \text{ psi (Max. Principal Tensile Stress)}$$

From Eq. (17):

$$\sigma_2 = \frac{\sigma_A}{2} - \sqrt{\left[\frac{\sigma_A}{2}\right]^2 + \tau_A{}^2}$$

$$\sigma_2 = \frac{-78,840}{2} - \sqrt{\left[\frac{-78,840}{2}\right]^2 + (57,338)^2}$$

σ_2 = -109,000 psi (Max. Principal Compressive Stress)

From Eq. (18):

τ_M = 72,533 psi (Max. Combined Shearing Stress)

From Eq. (22):

$$\sigma_1 = \sigma_B$$

σ_1 = 157,681 psi (Max. Principal Tensile Stress)

From Eq. (23):

σ_2 = 0 (Max. Principal Compressive Stress)

From Eq. (24):

$$\tau_M = \frac{\sigma_B}{2}$$

$$\tau_M = \frac{157,681}{2}$$

τ_M = 78,840 psi (Max. Combined Shearing Stress)

$$\text{S.F.} = \frac{\tau_Y}{\tau_M} = \frac{130,560}{78,840} = 1.66$$

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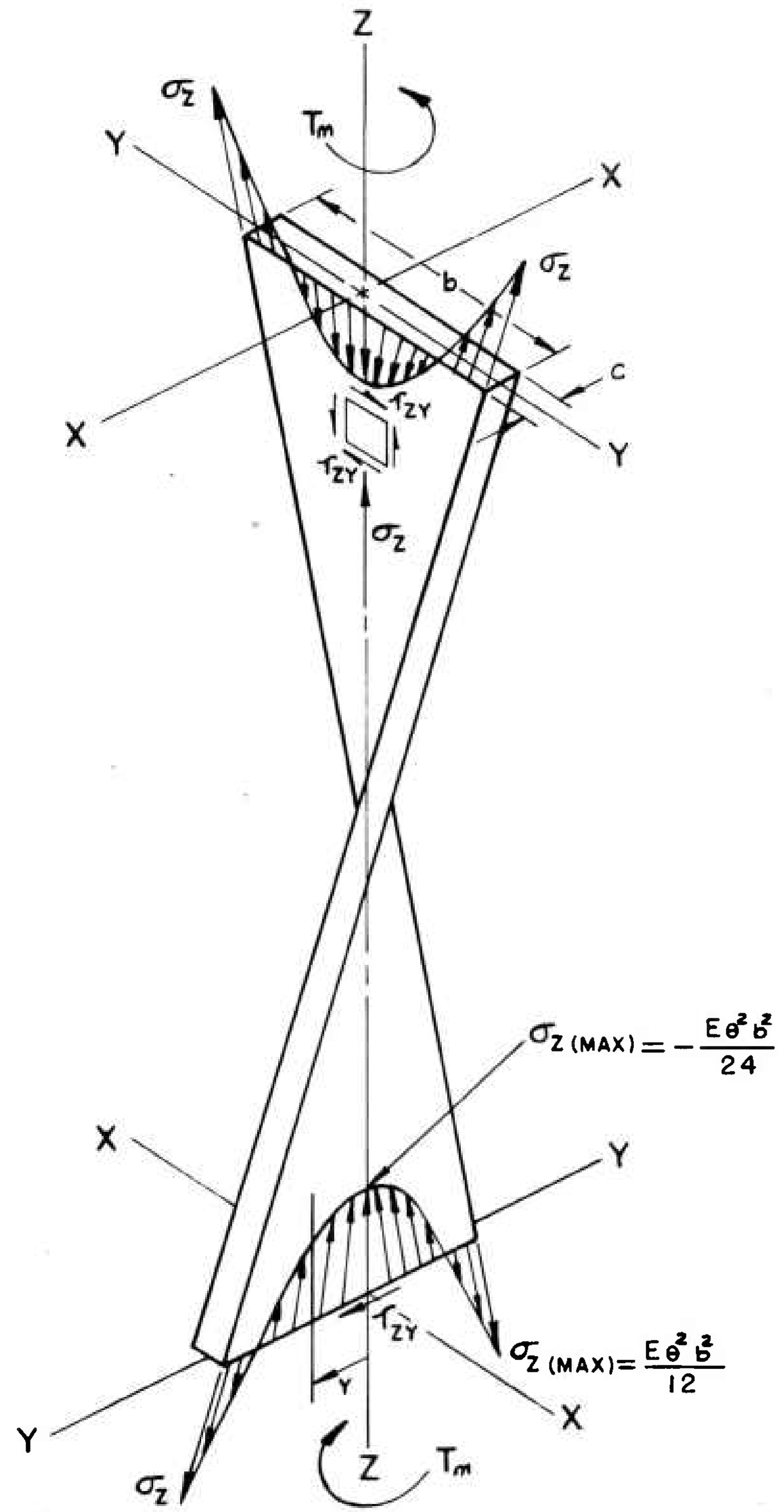
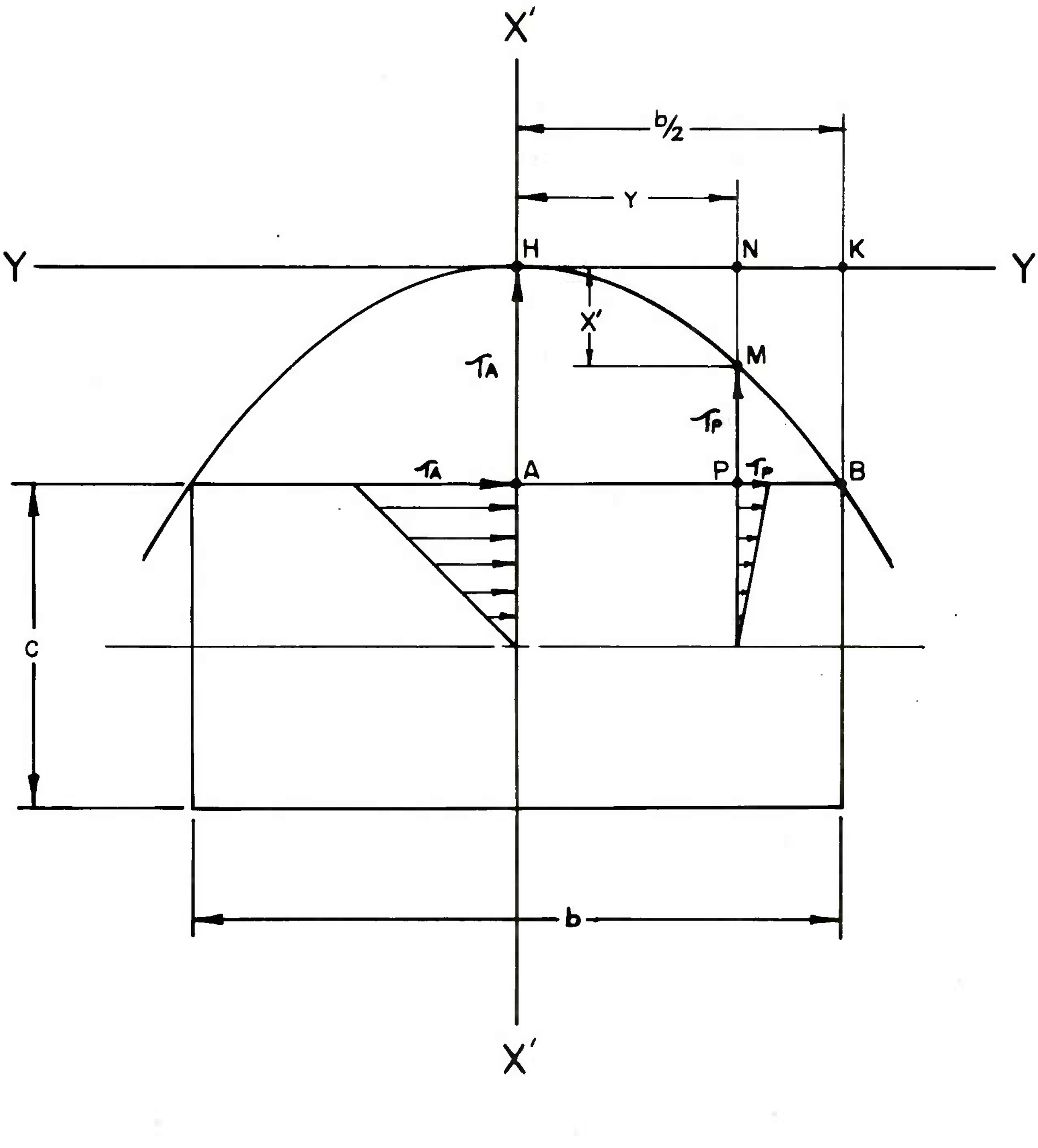


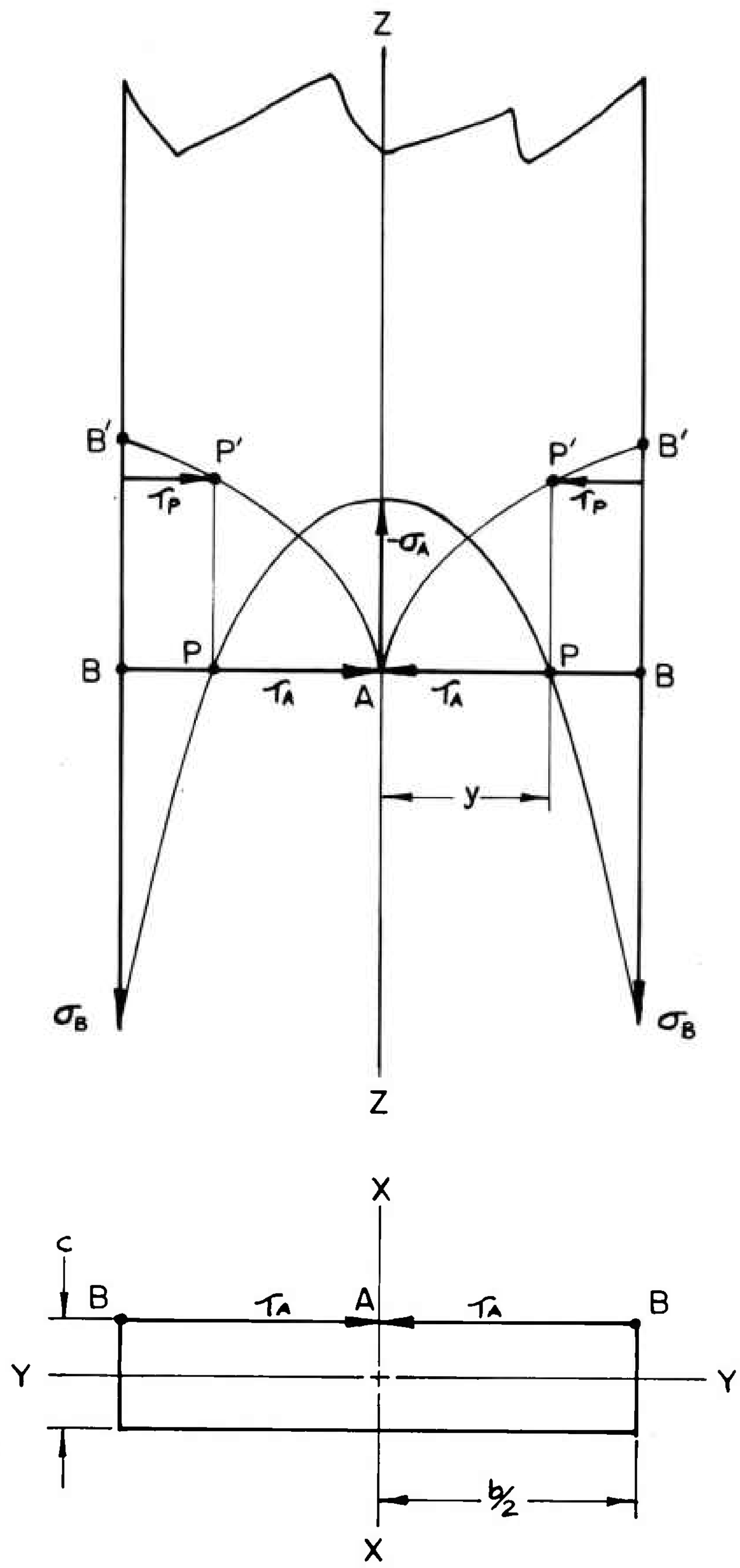
LONGITUDINAL STRESS IN LEAF SPRING UNDER TORSION

FIG. 1



Represpentation Of Torsional Shear Stress Across Strip Width

FIG. 2



SHEARING & LONGITUDINAL STRESS IN LEAF SPRINGS UNDER TORSION

FIG. 3

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